FACTOR INVESTING IN TIMES OF STRESS

DANIEL FANG, CFA, CAIA
Sr. Quantitative Research Analyst

SRI KANCHARLA, CFA
Head of Large Cap Quantitative Portfolio Management

ROB LEHNHERR, CFA
Head of Quantitative Equity Research

KEY FINDINGS

- We apply a form of unsupervised machine learning (Hidden Markov Model) to derive the probability of being in a stressed economic state. We use the probability to dynamically adjust the weights of a multi-factor strategy that employs a maximum Sharpe ratio optimization.

- An empirical analysis of the stocks in the MSCI World Index shows that our ML-enhanced dynamically-weighted strategy outperforms a baseline multi-factor strategy in both absolute and risk-adjusted terms over the most recent 27-year period.

SUMMARY

In this paper, we evaluate whether macroeconomic variables can inform conditional expectations of style factor performance to improve factor investing outcomes. We apply a Hidden Markov Model (HMM) to a composite economic indicator (U.S. Leading Economic Index) to derive the current probability of being in a stressed economic state. We show that the probability of stress as reported by the HMM contains information about future style factor return distributions. We then demonstrate by way of a case study how the performance of a baseline multi-factor strategy that is constructed with a maximum Sharpe ratio optimization can be improved by dynamically adjusting the factor weights in response to the HMM output. Over the 27-year simulation period of stocks in the MSCI World Index, the regime switching multi-factor timing strategy outperformed the baseline portfolio in both absolute (9.9% vs. 9.3% annualized return) and risk-adjusted terms (0.75 vs. 0.61 information ratio).

Factor investors commonly utilize long-term (strategic) estimates of risk and return to construct multi-factor portfolios. Although diversified factor strategies have been shown to achieve superior outcomes than single factor portfolios over a long horizon, the inability of fixed weighting schemes to respond to fast-changing market dynamics in the short run is a perceived weakness. The diversification benefit among factors is sensitive to economic conditions and therefore varies across the business cycle. **Exhibit 1** shows the average factor correlations of value, momentum, low volatility, and quality in the MSCI World Index over the last 28 years. Although average factor correlations have oscillated near zero since the global financial crisis, there has been a steady increase since 2021. When rising correlations are coupled with poor factor performance, even multi-factor strategies are susceptible to sustained periods of underperformance and steep drawdowns.

In recent years a body of research has emerged which explores the relationship between factor performance and macroeconomic conditions (see Amenc, Esakia, et al. (2019) and Boons (2016)). The application of tactical weighting schemes based on the prevailing economic environment (see Polk, Mo and et al. (2020)) is an area of research that is of particular interest to investors. Encouraged by these findings, we have designed a framework for detecting periods of heightened economic stress and tilting factor weights in response. A machine learning technique (Hidden Markov Model) was applied to classify macroeconomic regimes and derive the probability of being in a given economic state at any point in time. In this paper we introduce our framework and demonstrate its potential by way of a simple case study, in which a simulated dynamic factor strategy outperformed a traditional baseline strategy over the last 27 years in both absolute and risk-adjusted terms.

Exhibit 1: Rolling Three-Year Average Factor Correlations in the MSCI World Index (12/31/1995 – 12/31/2023)
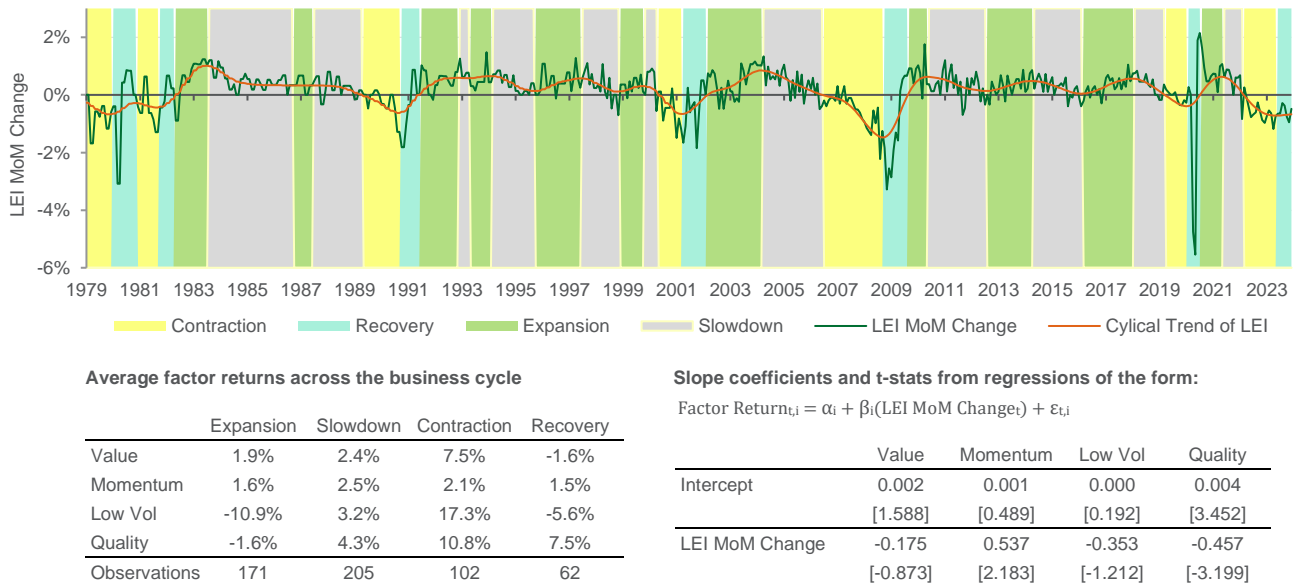

Source: Northern Trust Quantitative Research, MSCI, FactSet. Data from 12/31/1995 to 12/31/2023. Average long/short factor return correlations reported for value, momentum, low volatility, and quality. Top (30%) and bottom (30%) portfolios were market-cap weighted and rebalanced monthly. MSCI FaCS factor definitions used.

MACROECONOMIC INFLUENCE ON FACTOR PERFORMANCE

The influence of macroeconomic conditions on factor performance is widely recognized. A question we are often asked by investors is not whether factors are affected by the economy, but rather, to what extent? In our 2018 white paper "Contraction, Recovery & Growth: How Factors Performed", we evaluated this question empirically under four economic regimes: expansion, slowdown, recession and recovery. These classifications were derived by applying an HP filter[1] to the month-over-month changes in The Conference Board's Leading Economic Index (LEI),[2] and inferring the regime from the level and slope of the resulting trendline. This methodology provides a framework for quantifying the impact of the economic cycle on factor performance, the results of which are summarized in **Exhibit 2**.

Exhibit 2: U.S. Economic Regime Model and Russell 1000 Factor Analysis (12/31/1978 – 12/31/2023)

| | Contraction | | Recovery | | Expansion | | Slowdown | | LEI MoM Change | | Cylical Trend of LEI |

Average factor returns across the business cycle

	Expansion	Slowdown	Contraction	Recovery
Value	1.9%	2.4%	7.5%	-1.6%
Momentum	1.6%	2.5%	2.1%	1.5%
Low Vol	-10.9%	3.2%	17.3%	-5.6%
Quality	-1.6%	4.3%	10.8%	7.5%
Observations	171	205	102	62

Slope coefficients and t-stats from regressions of the form:

$$\text{Factor Return}_{t,i} = \alpha_i + \beta_i(\text{LEI MoM Change}_t) + \varepsilon_{t,i}$$

	Value	Momentum	Low Vol	Quality
Intercept	0.002	0.001	0.000	0.004
	[1.588]	[0.489]	[0.192]	[3.452]
LEI MoM Change	-0.175	0.537	-0.353	-0.457
	[-0.873]	[2.183]	[-1.212]	[-3.199]

Source: Northern Trust Quantitative Research, The Conference Board, FTSE Russell, MSCI, FactSet. Data from 12/31/1978 to 12/31/2023. The Russell 1000 Index was launched in 1984 with a data inception date of 1978. Long/short factor portfolio returns reported. Top (30%) and bottom (30%) portfolios were market-cap weighted and rebalanced monthly. MSCI FaCS factor definitions used.

When evaluating factor performance in the context of the regime model we find the results to be generally intuitive. In particular, low volatility (low vol) and quality both exhibit strong counter-cyclical performance, with positive average returns during contractionary regimes (table on the left), and negative betas to month-over-month changes in the LEI (regression model on the right). These results conform to expectations for "defensive" factors. While momentum shows no discernable difference in average returns across the four economic regimes, its positive beta to the monthly changes in the LEI suggests it is pro-cyclical on a shorter time horizon. Value, by way of its performance during contractions (as "growth bubbles" burst), appears to be mildly defensive on average.

The framework put forth does a reasonable job of answering the question "to what extent does the economy affect factor performance?" As such, it is clear from the results that if an investor is skilled at predicting economic regimes then style factors may be utilized to express their views within the equity market. However, whether the framework itself can be useful to investors to help inform their view is an altogether different question. While the LEI is much more responsive than traditional indicators such as GDP, the regime classifications derived by the model are backward-looking, and the reported regression results are contemporaneous (i.e., current period returns are regressed against current period changes in LEI). In order to evaluate the LEI as a timing indicator, we need a method to identify regimes on a point-in-time basis, and determine whether there is a relationship between the current regime and *future* returns.

CLASSIFYING THE CURRENT REGIME

While the HP filter introduced in the previous section is useful for business cycle analysis, the output may be too slow-moving to inform an investing decision. Furthermore, slope estimates of the cyclical trend are sensitive to the lambda (λ) parameter chosen, which may result in a false sense of precision when choosing among four distinct regimes. For these reasons, we employ a different algorithm for current regime classification – a Hidden Markov Model[3] (HMM) – and ask it

---

[1] See Hodrick and Prescott (1997) and the methodology notes in the appendix.
[2] The U.S. LEI was chosen as a parsimonious, forward-looking composite indicator. The index includes 10 underlying components, including both financial (3) and non-financial (7) timeseries (see https://www.conference-board.org/topics/us-leading-indicators).
[3] See Baum and Petrie (1966), Rabiner (1989), and Stamp (2004).

to differentiate between only two regimes ("states" in HMM nomenclature). A key feature of the HMM is the ability to identify the ("hidden") states on its own, therefore eliminating the need to classify regimes for the purpose of model training. Doing so would require us to make distinctions which may or may not be relevant. For example, whether an economy technically fell into recession or narrowly avoided one likely has no bearing on the market response. In this regard the HMM is employed as a form of unsupervised learning, where it has the freedom to identify the states on its own. By specifying two states, our expectation is that the HMM will approximate "normal" vs. "stressed" economic regimes (alternatively, expansion vs. contraction, risk-on vs. risk-off, etc.).

The components of our "Economic Stress" Hidden Markov Model are depicted in **Exhibit 3**. It consists of a transition probability matrix (P), state-specific emission probability distribution functions for month-over-month changes in the LEI (Q), and a transition process describing the sequence of hidden economic states. By using a reinforced learning process, the model arrives at a maximum likelihood estimate of P, Q, and the probability of being in either state at any point in time.
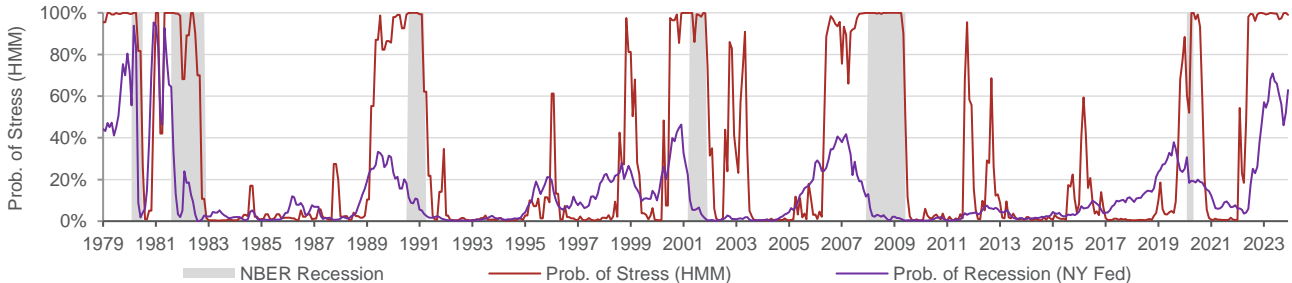
Exhibit 3: Components of the Economic Stress Hidden Markov Model (HMM)
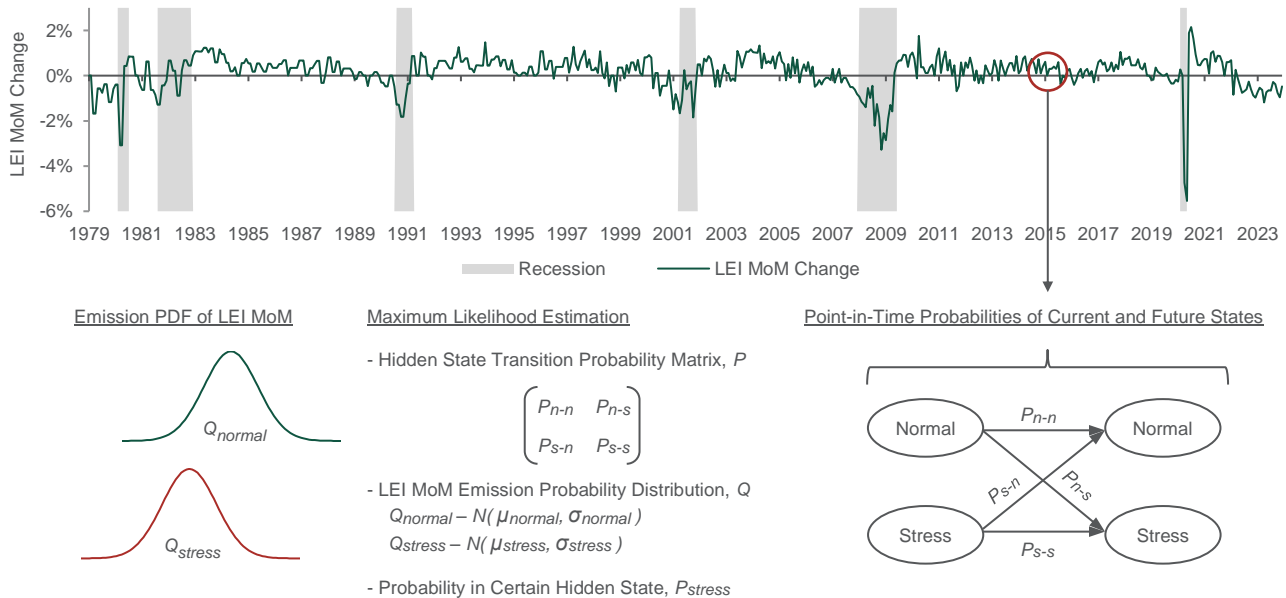
Source: Northern Trust Quantitative Research, The Conference Board, National Bureau of Economic Research. Data from 12/31/1978 to 12/31/2023.

In order to validate the model, we ran the HMM using point-in-time LEI data and captured the state probabilities at each month end[4] over the analysis period. **Exhibit 4** plots the "Probability of Stress"[5] from the HMM, official recession dates[6] from the National Bureau of Economic Research (NBER), and the probability of recession[7] from the Federal Reserve Bank of New York (NY Fed).

Exhibit 4: Hidden Markov Model Probability of Stress (12/31/1978 – 12/31/2023)

Source: Northern Trust Quantitative Research, The Conference Board, Federal Reserve Bank of New York, National Bureau of Economic Research. Data from 12/31/1978 to 12/31/2023.

---

[4] In order to avoid look-ahead bias, the HMM was independently estimated at every month end and LEI data was timestamped as of its published date. An expanding window was used for the LEI data beginning in 1959.

[5] We identified the HMM probability sequence that most closely aligns with known stress episodes for the purpose of labelling.

[6] See https://www.nber.org/research/data/us-business-cycle-expansions-and-contractions.

[7] See https://www.newyorkfed.org/research/capital_markets/ycfaq/.

Exhibit 4 reveals a clear association among the three variables charted, providing assurance that the HMM is indeed differentiating between "normal" and "stress" regimes as expected. Although the HMM is not immune to transient spikes, it demonstrates enough stability to indicate usefulness. During the time period reported (1978-2023), the probability of stress elevated above 50% a total of 16 times, with 10.5 months representing the average duration it remained above 50% (11 of the 16 periods were at least 3 months in length). As a practical matter, this level of persistence is required to mitigate excessive transaction costs assuming a breach of threshold induces portfolio repositioning. With a regime classification model now established, we proceed to evaluate its potential as a timing indicator.

CONDITIONING RETURNS ON (HIDDEN) STATE PROBABILITIES

If the regime model is to be useful for investors there must be a linkage between the probability of stress and future returns. To broaden the applicability of our analysis, we evaluated the HMM output (based on U.S. LEI[8] data) relative to the factor returns of the MSCI World Index. **Exhibit 5** plots the 3 month forward returns of the quality factor on the y-axis (left hand side (LHS)) corresponding to the probability of stress reported by the HMM on the x-axis, with each observation represented by a green diamond. Upon cursory review it appears the distribution of forward returns shifts upward as the probability of stress increases. To affirm this dynamic, we plot the average of the forward returns greater than the probability of stress (X) on the secondary y-axis (right hand side (RHS)). The steady increase in the dashed blue line confirms that the sample mean rises as the probability of stress increases. This result, when combined with the tendency for the probability of stress to remain elevated, provides support for the HMM as a potential timing indicator for the quality factor.

Exhibit 5: Quality Forward Return Distribution and Conditional Average (Probability of Stress > X) in the MSCI World Index (12/31/1995 – 12/31/2023)
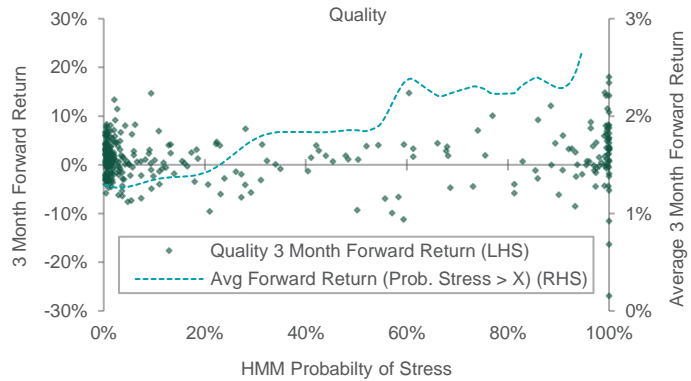
Source: Northern Trust Quantitative Research, The Conference Board, MSCI, FactSet. Data from 12/31/1995 to 12/31/2023. Long/short factor portfolio forward returns reported. Top (30%) and bottom (30%) portfolios were market-cap weighted and rebalanced monthly. MSCI FaCS factor definitions used.

Exhibit 6 extends this analysis by reporting conditional forward return averages and skew[9] for value, momentum, low volatility (low vol), and quality in the MSCI World Index (for more robust distribution analysis refer to Exhibit A in the appendix). Interestingly, upward trends in conditional averages for low volatility and quality are accompanied by downward trends in conditional skew for momentum, low volatility, and quality. This has implications for both risk and return – while the expected returns for defensive factors are increasing, so too are the chances for junk rallies and momentum crashes. This highlights the important distinction between macroeconomic conditions and market expectations, and the risks that can surface when the two decouple.

Exhibit 6: Conditional Forward Return Averages and Skew in the MSCI World Index (12/31/1995 – 12/31/2023)
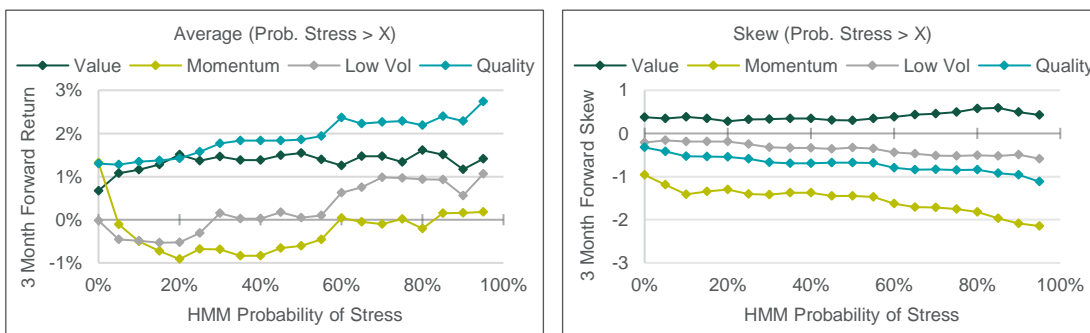
Source: Northern Trust Quantitative Research, The Conference Board, MSCI, FactSet. Data from 12/31/1995 to 12/31/2023. Long/short factor portfolio forward returns reported. Top (30%) and bottom (30%) portfolios were market-cap weighted and rebalanced monthly. MSCI FaCS factor definitions used.

[8] The U.S. LEI data has an extensive history for model training, and we believe it to be a reasonable proxy for global macro conditions.
[9] We omit conditional standard deviation here as it is increases with the probability of stress as one would expect. For more details refer to the appendix.

REGIME SWITCHING MULTI-FACTOR STRATEGY

In order to evaluate the ability of the HMM to improve multi-factor portfolios, we conducted a simple case study from the perspective of a global investor. To reduce complexity, the case study utilizes a long-only, top-down model to allocate to four factors – value, momentum, low volatility, and quality. In order to isolate the impact of the HMM, we first established a baseline model that represents a strategic multi-factor implementation. We then constructed a "multi-factor timing" strategy that modifies the parameters dynamically in response to the HMM output. Both the baseline and timing strategies were implemented as optimizations of the general form presented in **Exhibit 7**.

Exhibit 7: Multi-Factor Strategy Allocation Model

Maximize $(r^t w - \lambda w^t \Sigma w - \tau T)$

Subject to
$$w^t 1 = 1$$
$$w >= 0$$

Where
  r = vector of expected factor returns
  w = vector of factor weights
  λ = risk aversion parameter[10]
  Σ = factor covariance matrix[11]
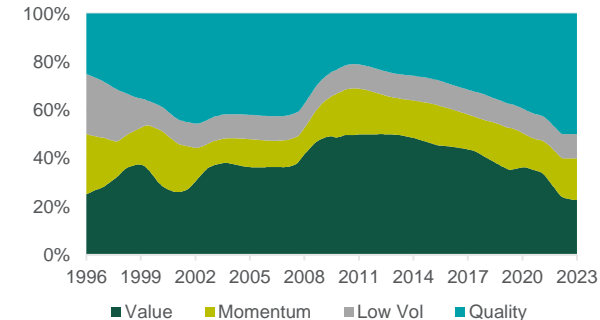 τ = transaction cost parameter[10]
  T = portfolio turnover[12]

With respect to the baseline model, the factor return assumptions represented the most important consideration. This decision was magnified by the persistent (negative) beta exposure embedded in the low volatility factor, which commonly results in it having the highest volatility among the factors. We ultimately chose to assign equal expected returns to all factors,[13] thereby reducing the first term of the objective function to a constant. This had the effect of biasing the baseline model away from low volatility (and to a lessor extent, momentum).[14] However, from the perspective of a benchmark-aware investor, we find that the preferred low volatilty allocation is often lower than other factors in order to maintain a portfolio beta near the benchmark. To ensure that the baseline model remained diversified, we constrained the factor weights such that $0.1 <= w <= 0.5$,[15] and assigned a 25% weight to each factor at the beginning of our simulation period.

Exhibit 8 charts the factor allocation of the baseline strategy over the 27-year analysis period[16] (12/31/1996 to 12/31/2023). Given that the expected factor returns were assumed to be equal, the composition of the baseline strategy was driven mainly by the covariance matrix. The influence of the risk aversion term of the objective function becomes clear when comparing the baseline allocation in Exhibit 8 to the time series of factor volatilities reported in Exhibit B of the appendix. In our experience, the factor allocations of the baseline strategy roughly resemble the output of an equal active risk contribution methodology – a popular approach for strategically allocating among factors.

With the baseline model established, we defined the timing strategy such that it responds to periods of heightened economic stress, using 50% as the threshold for delineating between "normal" and "stress" environments.[17] When the probability of stress breached 50%, the timing strategy a) overrode the expected returns of the baseline strategy and b) adjusted the constraints to employ factor tilts. Specifically, the timing strategy assigned average factor returns observed during recent stress periods for each individual factor,[18] and allowed up to a 20% factor tilt[19,20] from the baseline allocation. When the stress period abated (probability of stress fell below 50%), the timing strategy restored the parameters of the baseline model, and eventually converged to the baseline allocation (in accordance with the transaction cost term of the objective function).

Exhibit 8: Factor Weights of the Baseline Multi-Factor Strategy in the MSCI World Index (12/31/1996 – 12/31/2023)

Source: Northern Trust Quantitative Research, MSCI, FactSet. Data from 12/31/1996 to 12/31/2023. MSCI FaCS factor definitions used.

---

[10] The risk aversion (λ) and transaction cost (τ) parameters were set to 0.8 and 0.2 respectively.

[11] The (4x4) factor covariance matrix was computed with an expanding window of daily returns and a half-life of 1 year. In order to reduce the effects of parameter uncertainty, off-diagonal elements were scaled by 0.5.

[12] Represented by the sum of squared differences in factor weights (initial vs. optimal).

[13] Assigned by computing the rolling 10-year average return for each factor, and then averaging across all four factors.

[14] By contrast, had we assumed equal Sharpe ratios among the factors, this would have favored low volatility due to its higher volatility.

[15] This constraint is binding less than 1/3 of the time (97 of the 324 months in the simulation period).

[16] The simulation required a 1-year warmup period for the covariance matrix (12/31/1995 to 12/31/1996).

[17] Using this criterion there were 223 normal months and 101 stress months during the simulation period.

[18] The timing strategy averaged the last 60 monthly returns (i.e., last five years) belonging to a stress regime.
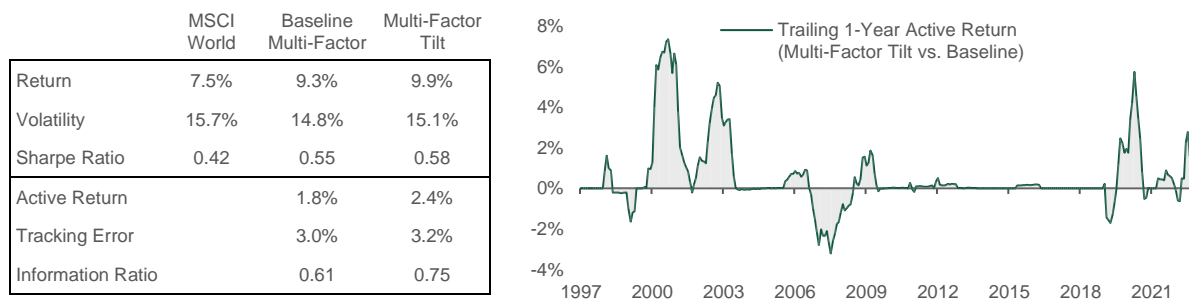
[19] Stress period constraints were specified as follows: $\max(0,(w^* - 0.2)) <= w <= (w^* + 0.2)$; where w^* = vector of baseline weights.

[20] This constraint was binding in 62 of the 101 stress months.

The performance summary of the MSCI World Index, baseline multi-factor strategy, and multi-factor timing strategy is reported in **Exhibit 9**, along with a trailing 1-year comparison between the baseline and timing portfolios.

Exhibit 9: Multi-Factor Strategy Performance and Trailing Active Return in the MSCI World Index (12/31/1996 – 12/31/2023)

	MSCI World	Baseline Multi-Factor	Multi-Factor Tilt
Return	7.5%	9.3%	9.9%
Volatility	15.7%	14.8%	15.1%
Sharpe Ratio	0.42	0.55	0.58
Active Return		1.8%	2.4%
Tracking Error		3.0%	3.2%
Information Ratio		0.61	0.75

Source: Northern Trust Quantitative Research, The Conference Board, MSCI, FactSet. Data from 12/31/1996 to 12/31/2023. Simulated multi-factor portfolio returns reported. Long-only portfolios were market-cap weighted and rebalanced monthly. MSCI FaCS factor definitions used. Past performance is no guarantee of future results. Index performance returns do not reflect any management fees, transaction costs or expenses. It is not possible to invest directly in any index.

The performance summary reported in the table on the left makes a strong case for multi-factor investing. When compared to the MSCI World Index, the baseline strategy delivered 180 bps of additional return (9.3% vs. 7.5%) at a lower volatility (14.8% vs. 15.7%), resulting in a significantly higher Sharpe ratio (0.55 vs. 0.42). Relative to the baseline portfolio, the regime switching multi-factor strategy added another 60 bps of return (9.9% vs. 9.3%), improving upon both the Sharpe ratio (0.58 vs. 0.55) and information ratio (0.75 vs. 0.61). The chart on the right provides some insight into the relative performance of the timing strategy vs. the baseline model. The HMM strategy did particularly well during the "tech wreck" in the early 2000s, appears to have mis-timed the global financial crisis (a large drawdown occurring before late 2008), and was well-positioned for the recent COVID-crisis (and aftermath). As it pertains to this hypothetical use case, we find the results of the regime switching strategy to be encouraging.

CONCLUSION

Factor cyclicality is often cited as the biggest challenge to factor investing. Factor diversification represents the most common method of mitigating cyclicality, but even multi-factor portfolios may be prone to periods of sustained underperformance or steep drawdowns. Recent research has shown that macroeconomic state variables can inform conditional expectations of risk and return to improve factor investing outcomes.

In this paper we applied a Hidden Markov Model to a composite economic indicator (U.S. LEI) to derive the current probability of being in a stressed economic state, and showed that it contained information about future factor return distributions. Specifically, average forward returns increased with the probability of stress for defensive factors (low volatility and quality), while the skew became more negative for low volatility, quality, and momentum. We then demonstrated by way of a simple case study how this information could be used to improve the risk-adjusted returns of a traditional multi-factor strategy. Over the 27-year simulation period in the MSCI World Index, the factor timing strategy added 60 bps of annualized return, and increased the information ratio from 0.61 to 0.75. We find these results to be encouraging, and believe this line of research has potential to increase alpha and improve (tail) risk management.

APPENDIX

Exhibit A: Conditional 3 Month Forward Return Distribution Properties in the MSCI World Index (12/31/1995 – 12/31/2023)

Prob. of Stress (X)		Value			Momentum			Low Volatility			Quality		
		25%	50%	75%	25%	50%	75%	25%	50%	75%	25%	50%	75%
Mean	< X	0.28%	0.30%	0.46%	2.45%	2.16%	1.75%	0.15%	-0.04%	-0.34%	1.15%	1.06%	0.98%
	> X	1.37%	1.54%	1.33%	-0.68%	-0.60%	0.02%	-0.31%	0.05%	0.97%	1.58%	1.86%	2.29%
	t-stat	-1.31	-1.44	-0.95	3.27	2.76	1.60	0.47	-0.09	-1.19	-0.77	-1.37	-2.12
	p-val	0.19	0.15	0.34	0.00	0.01	0.11	0.64	0.93	0.24	0.44	0.17	0.04
Volatility*	< X	6.0%	6.0%	6.3%	7.2%	7.3%	7.7%	7.1%	7.3%	7.4%	3.7%	3.7%	4.0%
	> X	9.2%	9.6%	9.8%	10.1%	10.6%	10.5%	10.9%	11.2%	11.6%	6.4%	6.8%	6.8%
	t-stat	20.81	24.05	18.55	13.73	14.60	6.59	15.31	13.04	12.08	21.08	28.11	18.99
	p-val	0.00	0.00	0.00	0.00	0.00	0.01	0.00	0.00	0.00	0.00	0.00	0.00
Skew	< X	0.16	0.15	0.05	0.13	0.06	-0.19	-0.01	-0.01	0.00	0.29	0.27	0.10
	> X	0.32	0.30	0.49	-1.41	-1.45	-1.75	-0.24	-0.33	-0.52	-0.59	-0.68	-0.85

*t-stats and p-values reported using Levene's test (W).

Source: Northern Trust Quantitative Research, The Conference Board, MSCI, FactSet. Data from 12/31/1995 to 12/31/2023. Long/short factor portfolio forward returns reported. Top (30%) and bottom (30%) portfolios were market-cap weighted and rebalanced monthly. MSCI FaCS factor definitions used.

Exhibit B: Factor Volatilities in the MSCI World Index (12/31/1996 – 12/31/2023)
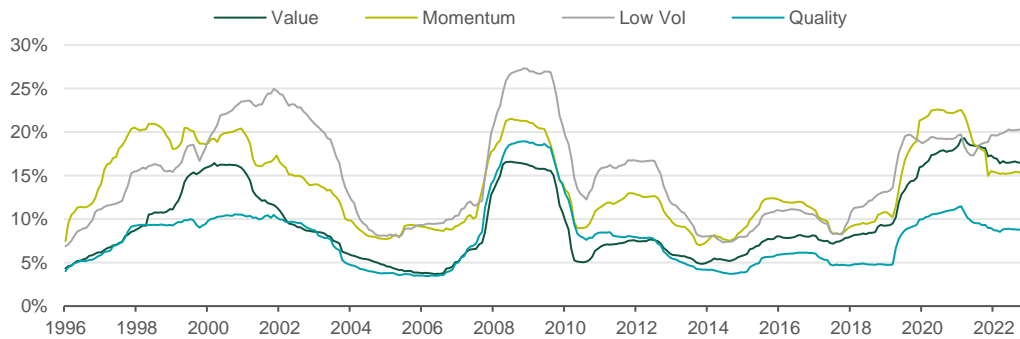
Source: Northern Trust Quantitative Research, MSCI, FactSet. Data from 12/31/1996 to 12/31/2023. Long/short factor portfolio volatilities computed with an expanding window of daily returns and a 1-year half life. Top (30%) and bottom (30%) portfolios were market-cap weighted and rebalanced monthly. MSCI FaCS factor definitions used.

Methodology Notes

Hodrick-Prescott Filter

The Hodrick-Prescott filter (HP filter) decomposes a time series y_t, into a trend component τ_t, a cyclical component c_t, and an error component ε_t, such that:

$$y_t = \tau_t + c_t + \varepsilon_t$$

Given a smoothing factor λ, there is a trend component that will solve:

$$\min_{\tau}\left(\sum_{t=1}^{T}(y_t - \tau_t)^2 + \lambda\sum_{t=2}^{T-1}[(\tau_{t+1} - \tau_t) - (\tau_t - \tau_{t-1})]^2\right)$$

Where
   y_t = input time series
   τ_t = trend component
   $c_t = (y_t - \tau_t)$ = cyclical component
   ε_t = error component
   λ = smoothing factor

The cyclical trend of the LEI computed by applying a Hodrick-Prescott filter (λ=100) to the monthly percentage change in The Conference Board U.S. Leading Economic Index.

References

Amenc, N., Esakia, M., Goltz, F., & Luyten, B. (2019). Macroeconomic risks in equity factor investing. The Journal of Portfolio Management, 45(6), 39-60.

Baum, L. E., & Petrie, T. (1966). Statistical inference for probabilistic functions of finite state Markov chains. The Annals of Mathematical Statistics, 37(6), 1554–1563.

Boons, M. (2016). State variables, macroeconomic activity, and the cross section of individual stocks. Journal of Financial Economics, 119(3), 489-511.

Hodrick, R., & Prescott, E. C. (1997). Postwar U.S. business cycles: An empirical investigation. Journal of Money, Credit, and Banking, 29(1), 1-16.

Hunstad, M., Dekhayser, J., & Fang, D. (2018). Contraction, recovery & growth: How factors performed. Northern Trust White Paper.

Polk, C., Haghbin, M., & De Longis, A. (2020). Time-series variation in factor premia: The Influence of the Business Cycle. Journal of Investment Management, 18(1).

Rabiner, L. (1989). A tutorial on Hidden Markov Models and selected applications in speech recognition. Proceedings of the IEEE, 77, 257-286.

Stamp, M. (2004). A revealing introduction to Hidden Markov Models. Department of Computer Science San Jose State, 1-20.

Wang, M., Lin, Y.H., & Mikhelson, I. (2020). Regime-switching factor investing with Hidden Markov Models. Journal of Risk and Financial Management, 13(12), 311.

Important Information

NTAM products and services are issued in the United Kingdom by Northern Trust Global Investments Limited, issued in the European Economic Association ("EEA") by Northern Trust Fund Managers (Ireland) Limited, issued in Australia by Northern Trust Asset Management (Australia) Limited (ACN 648 476 019) which holds an Australian Financial Services Licence (License Number: 529895) and is regulated by the Australian Securities and Investments Commission (ASIC), and issued in Hong Kong by The Northern Trust Company of Hong Kong Limited which is regulated by the Hong Kong Securities and Futures Commission.

This information is directed to institutional, professional and wholesale current or prospective clients or investors only and should not be relied upon by retail clients or investors. The information is not intended for distribution or use by any person in any jurisdiction where such distribution would be contrary to local law or regulation. NTAM may have positions in and may effect transactions in the markets, contracts and related investments different than described in this information. This information is obtained from sources believed to be reliable, its accuracy and completeness are not guaranteed, and is subject to change. Information does not constitute a recommendation of any investment strategy, is not intended as investment advice and does not take into account all the circumstances of each investor.

This report is provided for informational purposes only and is not intended to be, and should not be construed as, an offer, solicitation or recommendation with respect to any transaction and should not be treated as legal advice, investment advice or tax advice. Recipients should not rely upon this information as a substitute for obtaining specific legal or tax advice from their own professional legal or tax advisors. References to specific securities and their issuers are for illustrative purposes only and are not intended and should not be interpreted as recommendations to purchase or sell such securities. Indices and trademarks are the property of their respective owners. Information is subject to change based on market or other conditions.

All securities investing and trading activities risk the loss of capital. Each portfolio is subject to substantial risks including market risks, strategy risks, advisor risk, and risks with respect to its investment in other structures. There can be no assurance that any portfolio investment objectives will be achieved, or that any investment will achieve profits or avoid incurring substantial losses. No investment strategy or risk management technique can guarantee returns or eliminate risk in any market environment. Risk controls and models do not promise any level of performance or guarantee against loss of principal. Any discussion of risk management is intended to describe NTAM's efforts to monitor and manage risk but does not imply low risk.

Past performance is not a guarantee of future results. Performance returns and the principal value of an investment will fluctuate. Performance returns contained herein are subject to revision by NTAM. Comparative indices shown are provided as an indication of the performance of a particular segment of the capital markets and/or alternative strategies in general. Index performance returns do not reflect any management fees, transaction costs or expenses. It is not possible to invest directly in any index. Net performance returns are reduced by investment management fees and other expenses relating to the management of the account. Gross performance returns contained herein include reinvestment of dividends and other earnings, transaction costs, and all fees and expenses other than investment management fees, unless indicated otherwise. For U.S. NTI prospects or clients, please refer to Part 2a of the Form ADV or consult an NTI representative for additional information on fees.

Forward-looking statements and assumptions are NTAM's current estimates or expectations of future events or future results based upon proprietary research and should not be construed as an estimate or promise of results that a portfolio may achieve. Actual results could differ materially from the results indicated by this information.